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The octave potencies convention: a mathematical model of dilution and succussion

David J Anick*

Harvard Medical School, McLean Hospital, Centre Bldg. 11, 115 Mill St., Belmont, MA 02478, USA

Several hypothesized explanations for homeopathy posit that remedies contain a concentration of discrete information-carrying units, such as water clusters, nanobubbles, or silicates. For any such explanation to be sustainable, dilution must reduce and succussion must restore the concentration of these units. Succussion can be modeled by a logistic equation, which leads to mathematical relationships involving the maximum concentration, the average growth of information-carrying units rate per succussion stroke, the number of succussion strokes, and the dilution factor (x, c, or LM). When multiple species of information-carrying units are present, the fastest-growing species will eventually come to dominate, as the potency is increased.
An analogy is explored between iterated cycles dilution and succussion, in making homeopathic remedies, and iterated cycles of reseeding and growth, in bacterial cultures. Drawing on this analogy, the active ingredients in low and medium potency remedies may be present at early dilutions but only gradually come to 'dominate', while high potencies may develop from the occurrence of low-probability but faster-growing 'mutations.' Conclusions from this model include: 'x' and 'c' potencies are best compared by the amount of dilution, not the amount of succussion; the minimum number of succussion strokes needed per cycle is proportional to the logarithm of the dilution factor; and a plausible interpretation of why potencies at approximately regular ratios are traditionally used (the octave potencies convention). *Homeopathy* (2007) **96,** 202–208.

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Introduction

Homeopathic remedies are made by iterated dilution (in water or ethanol-water) and succussion (vigorous repeated pounding of the closed vial against a firm surface), starting from a mother tincture ('MT'), most often a plant or animal extract. Hahnemann experimented mainly with 1:9 ('x'), 1:99 ('c'), and 1:50 000 ('LM') dilutions. These have become, by convention, the dilution ratios that are used in commercially available remedies. We will call the volume increase during dilution the 'dilution factor' and denote it as H. Thus, $H = 10$ for 'x' remedies, $H = 100$ for 'c' remedies, and $H = 50\,001$ for 'LM' remedies.

The number of dilution-succussion cycles is the potency of the remedy, denoted P. Within homeopathic practice, while it is theoretically possible to give a patient any potency of a remedy, only certain potencies are normally available and stocked. For the 'x' series these are the '6', '12', '30', and '200' potencies, while for the 'c' series one can get '6', '12', '30', '200', '1000', and '10 000'. Although in other homeopathic traditions different series may be used, there is a similar progression. LM's start with LM1 and every potency is available (i.e. LM2, LM3, LM4, etc.) up to LM10 or so. The potencies most frequenty dispensed

*Corresponding author. DJ Anick, Harvard Medical School, McLean Hospital, Centre Bldg. 11, 115 Mill St., Belmont, MA 02478, USA.
E-mail: david.anick@rcn.com
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in practice (at least in the Anglo-American tradition), by far are the 6c and 12c ('low potencies'), 30c and 200c ('medium potencies'), and 1000c and 10 000c ('high potencies'). Homeopaths generally believe that remedies gain strength with more dilution-succussion cycles, although there are believed to be qualitative differences: 'stronger' is not necessarily 'better'. Posology, or how to decide what potency to give, is a complex subject about which there are many theories. In general, higher potency remedies are used when the remedy choice is more certain, when the patient's vital force is stronger, and when the problem is chronic rather than acute.

Is there any rationale for the sequence: 6, 12, 30, 200, 1000, 10 000? The sequence bears some resemblance to a geometric progression, and the use of fixed potencies with (supposedly) approximately equal ratios is called the 'Octave potencies convention' (OPC). I wondered, could there possibly be a rationale for the OPC? The usual thinking about this is that the remedy's qualities change gradually with potency, eg a 12c and a 13c are nearly the same, and 13c and 14c are nearly the same, but enough small changes accumulate in going from 12c to 30c, that 30c may bring different results in the clinic from 12c. While a 12c and a 13c are 'nearly the same', a 1000c and a 1001c would be considered to be clinically interchangeable.

Various hypotheses have been put forward to 'explain' homeopathy in terms of conventional physics and chemistry. 'Local' hypotheses posit that remedies differ from untreated water in that they contain a population or concentration of an active ingredient. For some explanations, the active ingredient is a (hypothetical) persistent structural feature in what is chemically pure water, such as a zwitterion,[1] a clathrate,[2] or nano-bubble.[3] The 'silica hypothesis' posits that SiO_2 derived from the glass walls of the succussed vials is condensed into remedy-specific oligomers or nanocrystals, or else that silica nanoparticle surface is modified in patches to carry remedy-specific information.[4]

The mathematical model developed here is compatible with any of these explanations. Let Q denote the concentration of 'active ingredient'. Depending on the hypothesis, Q could be the concentration of a particular zwitterion, of a particular species of nano-bubble, of a particular silica oligomer (or family of oligomers), or of a specific silica nanoparticle surface feature. Note that the concentration of active ingredient in ordinary solvent is zero or is assumed to be negligible. Right after dilution, the concentration will be $Q_{dil} = Q/H$.

The fundamental assumption underlying our mathematical model is the following. Since a 1000c and 1001c are (essentially) identical, we assume that the effect of diluting a remedy of concentration Q, followed by succussion, is to regenerate (approximately) the same concentration Q of the same active ingredient. The model will shortly be made more complex by postulating multiple species of active ingredients, but let us start with the assumption of a single active ingredient. Then succussion must raise the concentration from Q_{dil} back up to $Q = HQ_{dil}$. If succussion did not raise the concentration by a factor of (on average) H, then after repeated cycles the concentration would dwindle to zero.

Modeling succussion

How does succussion raise the concentration by a factor of H (typically $H = 100$)? The answer depends on what the active ingredient is alleged to be. For the nano-bubble hypothesis, a nano-bubble might, during the pressure wave of succussion, organize the adjacent H_2O into another copy of the same nano-bubble, and both bubbles might survive as structural features after the pressure wave passes.

For the silica hypothesis, silica might be released into solution as $Si(OH)_4$ monomers by the mechanical agitation of succussion, and the specific silica nanocrystals might catalyze the formation of more copies of themselves out of the newly released monomers. It is beyond the scope of this article to assess or justify whether such notions are plausible.

Our starting point is to suppose that if any local hypothesis for homeopathy is valid, then there is some mechanism by which some structural feature replicates itself when succussed. We do not need to know what the feature is, or how it makes more copies, to develop the model.

Succussion consists of a series of 'succussion strokes'. During each stroke several things happen: pressure rapidly surges then returns to 1 atm, the solution is turbulently mixed with air, $Si(OH)_4$ enters solution, and so on. Let S denote the number of strokes used in each cycle. We postulate that in the course of S strokes, the concentration climbs from Q_{dil} to HQ_{dil}. We cannot say what happens during a single stroke since we do not know the specific mechanism, but the hypothesized mechanisms suggest that each unit (ie each zwitterion, each nano-bubble, each silica nanocrystal, etc.) uses the added 'raw material' (ie the added water or newly dissolving air or $Si(OH)_4$ monomers) to create more copies of itself. Thus, we assume that succussion strokes induce replication of the active units.

To call it 'replication' suggests a 2-for-1 process, but the process may not be 100% efficient. Instead of 2-for-1 we postulate that one succussion stroke raises the concentration of active units by a factor we call R. If Q_m is the concentration after m strokes with $Q_0 = Q_{dil}$, then $Q_1 = RQ_0$, $Q_2 = RQ_1$, and so on. This cannot continue forever, or Q_m would blow up exponentially. Replication ceases when the solution runs out of usable raw material. For instance, if the units are nano-bubbles, there will be some limit on how closely they can crowd together, and once the population reaches

the crowding limit they will not be able to replicate further. This situation is a familiar one in population biology: growth starts exponentially but then is capped by a finite carrying capacity. Mathematically it is modeled by assuming the actual growth rate is proportional to the amount of raw material accessible for further growth, which in turn is proportional to the difference between Q and a maximum concentration C. We obtain the discrete logistic equation,

$$Q_{m+1} - Q_m = (R-1)Q_m(C - Q_m)/C. \quad (1)$$

This equation does not have a simple solution in its discrete form, but the very similar equation

$$Q_{m+1} - Q_m = (R-1)Q_m(C - Q_{m+1})/C \quad (2)$$

has the very nice exact solution

$$Q_m = \frac{CR^m}{R^m - 1 + C/Q_0}, \quad (3)$$

which exhibits the expected S-shaped curve asymptotic to C as $m \to \infty$. After S succussion strokes the concentration is HQ_0, ie $Q_S = HQ_0$, and putting this into Eq. (3) shows that the concentration at the end of each cycle is given by

$$Q_S = C\frac{R^S - H}{R^S - 1}. \quad (4)$$

According to Eq. (4), if $R^S \gg H$, then Q_S will be close to the maximum allowable concentration C, but if $R^S < H$, there is no (positive) solution, and the concentration will die out to zero with repeated dilution–succussion cycles.

This already tells us something interesting about the number of succussion strokes needed. If our growth rate reflects 'perfect' replication when very dilute, ie $R = 2$, then to get $R^S > H$ we require a minimun of 7 succussion strokes per cycle for $H = 100$ (since $2^7 > 100$ but $2^6 < 100$), and a minimum of 16 strokes for the LM series. For a slower growth rate like $R = 1.2$, we need at least 38 strokes per cycle to bring the concentration u to 90% of the maximum when $H = 100$, and 72 strokes per cycle for LM's. (These stroke counts are obtained by setting $Q_S/C = 0.9$ in Eq. (4) and solving for S).

Although we have no experimental evidence to give us a range for R, Eq. (4) suggests that we should not skimp on succussion, with 40 strokes as a reasonable minimum when making 'c' potencies. Hahnemann himself held changing views about the optimum value for S. In the 5th edition of the Organon he recommended $S = 2$ but revised the figure upward to $S = 100$ in the 6th edition [5, p. 270].

Two active ingredients

If there were just a single active ingredient, dilution would reduce and succussion would restore the concentration each cycle. Nothing would change with dilution–succussion cycles and there would be no point in repeating dilution and succussion. But suppose there are two active ingredients, each of which would, if it were alone, increase according to Eq. (1). Approximate Eq. (1) by a continuous version, with the stroke count parameter 'm' being replaced by a 'time' parameter t. The difference equation (1) becomes the familiar logistic differential equation,[6]

$$X' = \frac{dX}{dt} = rX(1 - X), \quad (5)$$

where we have scaled the concentration so that $X = Q/C$, and instead of R we encounter $r = \ln(R)$. The solution is $X(t) = (1 + (X(0)^{-1} - 1)e^{-rt})^{-1}$, which is the continuous form of Eq. (3).

Let us add a second species of active ingredient, eg a different nano-bubble type or a different form of silica crystal. Let us assume that when some of each is present, the two species ignore each other. Each species replicates at its own rate as if the other were not present. There is still interaction, however, since both species draw upon the same raw material, of which there is a fixed amount. This sets up a competition scenario. The differential equations are

$$\begin{cases} X' = rX(1 - X - Y), \\ Y' = sY(1 - X - Y), \end{cases} \quad (6)$$

where without losing generality we assume $s > r$. There is no elementary solution but the trajectories can be found by dividing the two equations, giving $dY/dX = (s/r)(Y/X)$, hence

$$Y/Y(0) = (X/X(0))^{s/r}. \quad (7)$$

Let us further assume that the number of succussion strokes is large enough that the limiting concentrations are nearly attained; this is modeled by letting $t \to \infty$. Then the final concentrations are given by the intersection of trajectory (7) with the line $1 - X - Y = 0$.

Suppose we conduct a series of dilution–succussion cycles for this two-component model. Let (X_P, Y_P) describe the concentrations at the end of the Pth cycle, P denoting the potency. The relationship between (X_{P+1}, Y_{P+1}) and (X_P, Y_P) is as follows. Starting with (X_P, Y_P), after dilution the concentrations are $(X_P/H, Y_P/H)$. Putting $X(0) = X_P/H$ and $Y(0) = Y_P/H$ into Eq. (7), we see that (X_{P+1}, Y_{P+1}) is found by intersecting the line $X + Y = 1$ with the curve $HY/Y_P = (HX/X_P)^{s/r}$.

To proceed it is easier to work with the 'pH' values, $x = -\log(X)$ and $y = -\log(Y)$ ('log' is $\log_{10}$). Set $h = \log(H)$ (so $h = 2$ for 'c' potencies). Referring to Figure 1, dilution takes us on a line of slope 1 from (x_P, y_P) to $(x_P + h, y_P + h)$, and succussion takes us in a straight line of slope s/r from there back to the curve $10^{-x} + 10^{-y} = 1$. (x_{P+1}, y_{P+1}) is the intersection of that curve and line.

Iterating the process, we 'walk' along the curve, at some point transitioning from values where $y > x$ (meaning that $X > Y$ and 'X' is the dominant species present) to values where $x > y$ (ie 'Y' dominates). After

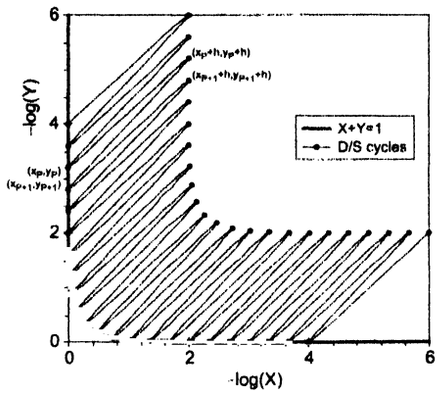

Figure 1 Log concentrations in alternating succussed and diluted stages of a two-ingredient remedy undergoing a transition from '*X*'-dominated to '*Y*'-dominated, for $s/r = 1.2$. Succussed remedies lie on the blue curve, $10^{-x}+10^{-y} = 1$ ($X+Y = 1$). Dilution raises both x and y by $h = 2$.

the transition $x_P \to \infty$ while $y_P \to 0$, ie '*X*' continues fade to zero while '*Y*' converges to the maximum concentration. Before the transition, ie where $y > x$, the curve $10^{-x} + 10^{-y} = 1$ is nearly vertical and a good approximate formula relating (x_{P+1}, y_{P+1}) to (x_P, y_P) is

$$x_{P+1} \approx x_P, \quad y_{P+1} \approx y_P + h(r-s)/r \tag{8a}$$

while after the transition (where $x > y$) it is nearly horizontal and

$$x_{P+1} \approx x_P + h(s-r)/s, \quad y_{P+1} \approx y_P. \tag{8b}$$

Using only the fact that the curve $10^{-x} + 10^{-y} = 1$ has a negative slope, we obtain the inequalities

$$h(s-r)/r > (x_{P+1} - y_{P+1}) - (x_P - y_P) > h(s-r)/s. \tag{9}$$

Clearly, what happens with increasing potency is that the slower-growing species '*X*' is gradually replaced by the faster-growing species '*Y*'. Exponentiating Eq. (9) we see that the concentration ratio Y_P/X_P increases by a factor of between $10^{h(s-r)/s}$ and $10^{h(s-r)/r}$, or between $H^{(s-r)/s}$ and $H^{(s-r)/r}$, with each dilution-succussion cycle. Pre-transition the ratio increase is very close to $H^{(s-r)/r}$, while post-transition it is very close to $H^{(s-r)/s}$. Thus, the transition potency can be predicted fairly easily if one knows the growth rates and the initial concentration ratio at a low potency. If s/r is only slightly bigger than 1, it takes more cycles to reach the transition and the transition occurs gradually over several cycles. If s/r is substantially bigger than 1, the transition is reached quickly and occurs abruptly. Of course, there is no transition at all if the initial concentration of '*Y*' exceeds that of '*X*': in this case the slower growing '*X*' just declines, outcompeted by '*Y*'.

Translating this to the clinical context, the implication is that, remedies where the two-component model applies will feature one species below the transition potency, and a different species above it. For example, if the transition occurs at $P = 20$, then potencies below 20c should all have approximately the same clinical action, since the are all dominated by the same pre-transition active species, whereas those above 20c will be similar to each other but different from the pre-transition potencies. Because of this, having any one pre-transition remedy and any one post-transition remedy should suffice in the clinic. Having a '12c' and a '30c' would cover it.

The number of cycles needed to get from a potency whose concentration ratio is $W_P = Y_P/X_P$ to the transition potency, is about $-\log(W_P)/(h(s-r)/r)$. Without needing to know any values for s, r, or W_P, this formula tells us that the number of cycles needed is inversely proportional to $h = \log(H)$. Starting from the same point, 'c' potencies attain the transition twice as fast as '*X*' potencies, and 'LM' progress faster than 'c' by a factor of $\log(50\,001)/\log(1\,00) = 2.35$. More generally, our formulas show that each 'c' dilution-succussion cycle has almost exactly the same effect as two '*X*' cycles. To the extent that this type of model turns out to be valid, it appears to answer the long-standing argument in homeopathy as to whether dilution or succussion matters more in 'potentizing' remedies. This model predicts that it is the total amount of dilution that determines a remedy's properties. Succussion at each stage must exceed a minimum threshold, but succussing significantly beyond that threshold will not make much difference.

Our mathematical model of two structural 'species' with different growth rates competing for raw material and limited by a maximum concentration has a perfect analogy in population biology. The analogy would be two living species that compete for a resource base but one reproduces faster than the other. A series of cycles occur, driven by periodic natural disasters that decimate each species' numbers by the same factor of H each time. As they recover between disasters, the faster-growing species gains some ground each cycle and eventually replaces the slower-growing one.

Bacteriologists use this model deliberately to select for variants with desired traits. Bacteria with resistance to a toxin T will be 'faster-growing' in the presence of T. A baseline low mutation rate means that some low initial concentration of the bacteria is of the T-resistant 'species' (not necessarily a distinct species in the biological meaning). After culturing it to maximum growth with T, a small amount (eg 1%, corresponding to $H = 100$) is re-seeded onto a new dish and then recultured. After many cycles the T-resistant species comes to dominate. 'Dilution' is like seeding a sterile culture dish while 'succussion' is like growth and selection.

Multiple active ingredients

The model can be extended to n species of active ingredient, $n > 2$. The concentration of the ith species is denoted X_i, or if we also include the potency in the

notation, as $X_{i,P}$. The growth rate of X_i is in (R_i), and $-\log(X_i)$ is denoted x_i. The system of equations governing succussion is

$$X_i' = r_i X_i \left(1 - \sum_{j=1}^{n} X_j\right), \quad i = 1, \ldots, n. \tag{10}$$

We omit details of its solution. The effect of one dilution–succussion cycle is described by

$$x_{i,P+1} \approx x_{i,P} + h(r_{\mathrm{DOM}} - r_i)/r_{\mathrm{DOM}}, \tag{11}$$

where r_{DOM} denotes the growth rate of whatever species happens to have the greatest concentration at potency P. Note that Eq. (11) reduces to Eqs. (8a,b) when $n = 2$. The effect of one dilution–succussion cycle on the concentration ratio for any two of the species, say for $X_{i,P}/X_{j,P}$, is to change the ratio by a factor of $H^{(r_i - r_j)/r_{\mathrm{DOM}}}$, ie

$$(X_{i,P+1}/X_{j,P+1})/(X_{i,P}/X_{j,P}) \approx H^{(r_i - r_j)/r_{\mathrm{DOM}}}. \tag{12}$$

Depending on their initial concentrations, several of the n species may dominate in turn, but as $P \to \infty$, eventually only the fastest-growing species remains.

Figure 2 illustrates the model with $n = 4$ species and $H = 100$. We suppose that the four species are present at the 4c potency, having been generated by some process that utilizes components from the MT. Perhaps compounds in the MT might catalyze the formation of specific silicates through directed polymerization of $Si(OH)_4$ monomers. Again, how the MT and early potencies would do this is not relevant to our model. Initial (ie in the 4c potency) concentrations and growth rates (R_i) are taken to be: $X_1 = 0.99$, $R_1 = 1.2$; $X_2 = 0.01$, $R_2 = 1.3$; $X_3 = 10^{-8}$, $R_3 = 1.35$; $X_4 = 10^{-12}$, $R_4 = 1.36$. These are entirely made-up numbers but they are not implausible. Note that initial concentrations correlate inversely with growth rates. As a result we can expect that each species may lead the 'race' for an interval of several potencies, but ultimately X_4 will 'win.'

Figure 2 displays $\log(X_{i,P})$ as a function of P. Figure 2 was generated by a computer program that used the four-species analog of Eq. (1) to compute exact predictions of concentrations using $S = 40$ succussion strokes per cycle. Each of the first three concentrations dominates for a while but then at a transition gives way to the next faster-growing species. Transitions correspond to points where the top two curves cross: at $P = 6.5$, 23.5, and 79. With a log scale for the ordinate, each curve consists of a succession of nearly straight line segments. This behavior is explained by Eq. (11), which predicts that the slope should change at transition points (where r_{DOM} changes) but should remain nearly constant between transition points.

Figure 3 shows the same information but displays $X_{i,P}$ as a function of $\log(P)$. Note that for each of '6c', '12c', '30c', and '200c', a different species is dominant. Vertical lines have been added at these positions. Potency intervals are defined by which species dominates, and the potencies falling within any interval would be expected to be clinically equivalent. Interval boundaries occur at transition points: in this example the intervals are 4c to 6c, 7c to 23c, 24c to 79c, and 80c and up. Thus, there are just four essentially different remedies derivable from this MT.

Figure 3 illustrates the 'best case scenario' for the octave potencies convention: there are four species, each of which dominates one interval of potencies, and the potencies efficiently make available one potency from each interval. (For this MT, all potencies beyond 200c would be virtually identical to the 200c potency and would be unnecessary.) This illustrates what one would ideally want from a prescribing convention: one example of each dominant species is included, without redundancy. Given that the number of species and their growth rates must vary from MT to MT, it would be inconceivable that one number sequence (ie 6, 12, 30, 200, 1000, 10 000) would work in this ideal manner for every MT. Still, it may do a good enough job of balancing the need for simplification against comprehensive coverage, for the majority of MT's.

High potencies

Dr JT Kent, developer of the octave potencies concept, [7,8] actually continued the sequence beyond 10 000: the continuation was 50 000, 10^5, 5×10^5, 10^6. These 'very high' potencies are not often used today.

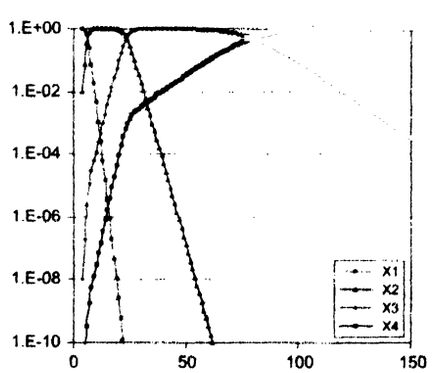

Figure 2 Log (conc) *vs* potency, for four-component model.

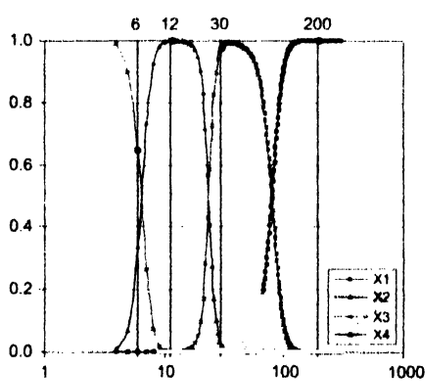

Figure 3 Conc *vs* log(potency), for four-component model.

Does our model support a role for high (1000 and 10 000) and very high potencies? As we have noted, use of a potency above 200c only makes sense if there is a transition that occurs at a potency higher than 200, and likewise a potency above 1000c adds something new only if there is a species whose transition to dominance occurs above $P = 1000$.

In Figure 2, the last transition (at 79c) resulted from two growth rates that are very close ($R_3 = 1.35$ and $R_4 = 1.36$), along with a very small initial concentration of X_4 (10^{-12}). For the model to yield a transition beyond 200c, there would have to be an even smaller difference in growth rates or a much smaller initial concentration (of the species whose transition to dominance occurs beyond 200c).

Tiny differences in rates are certainly a mathematical possibility, but this strikes me as unlikely to be the explanation for the majority of high potency remedies. Tiny initial concentrations likewise work in the model, but if we go below 10^{-17} or so we run into the Avogadro limit. (Concentrations have been scaled so that the maximum concentration C of a structural component is set to '1'. Measurements of silica[9,10] and other considerations place C in the micromolar range. If C is 10 μM then $X_P = 10^{-17}$ means 10^{-22} M, and in a 10 mL sample there would be 10^{-24} mol, ie probably none, of this active ingredient.)

There is a way around this, and that is to suppose that the species with the late (ie >200) transition is not present in the initial low-potency mix at all: it only appears later in the potentizing process, presumably as a result of imperfect replication of one of the other species during a succussion step. Drawing on the biological analogy, the late-transitioning species would arise as a mutation of an earlier-transitioning species. If the mutation rate is low, it could take many cycles of dilution and succussion until the mutation first appears. To survive, the mutation would need to have a selective advantage (which in our model means a faster growth rate).

If this is correct, the high potency remedies (and possibly some 30c's or 200c's as well) feature an active ingredient that arises out of a lower-potency active ingredient and eventually replaces it. Ballpark numbers might be that the mutation has only a 0.5% chance of arising in any given succussion–dilution cycle, and once it arises it takes 50 cycles to become dominant. Many of the cycles between 200c and 1000c may be doing nothing except 'waiting' until the chance event of this particular mutation occurs. However, with enough repetitions even a 0.5% event is almost sure to occur eventually. It has a $1-(0.995)^{750} = 97.6\%$ chance of occurring somewhere between the 200th and 950th cycle, and of achieving dominance between the 250th and 1000th cycle. According to this explanation, high potency remedies contain their intended active ingredient only with a certain probability, though the probability may be quite high (over 95%). The explanation for the need for a 10 000c would be that it depends upon the emergence of an even lower likelihood mutation (around 0.05% occurrence rate per cycle), and so on for the very high potencies.

Conclusion

Kent's octave potency sequence is widely accepted in homeopathic practice. In the clinic, when homeopaths refer to 'the next higher potency after 30c', they mean 200c, not 31c. Our model suggests a reason this may be literally correct: the 31c is essentially identical to 30c, but somewhere between 30c and 200c a transition occurs to the 'next' active ingredient. One cannot derive Kent's specific potency list from the model, but it does support Kent's principle of stocking discrete potencies that occur at approximately geometric ('octave') intervals.

We started with a single assumption, namely that each succussion stroke amplifies the structural active ingredient by drawing upon finite resources (space, H_2O, $Si(OH)_4$, or silica surface). This assumption led to a relationship (Eq. (4)) among the growth rate, dilution factor, and stroke count. Based on Eq. (4) we recommended a minimum of 40 succussion strokes per cycle, for 'c' potencies.

When there are multiple species of active ingredients with different growth rates, we assumed there was no interaction other than competition for the finite resources. The choice of language was intentional, to draw attention to a parallel in mathematical biology. This assumption can be questioned or altered. For example, there could be other interactions including cooperative ones between the species. Also, instead of a small number of distinct species there could be a continuum or near-continuum of species (eg nanobubble or nanocrystal size might be a continuous parameter) that is better handled with a diffusion–selection model.[11] A 'gradual evolution' derived from selection among a near-continuum of homeopathically active silicates has been hypothesized.[4] Our assumption of a small number of distinct active ingredients leads to a picture that in general is like Figures 2 and 3: most potencies contain a single 'dominant' species with the other species occurring at levels one or more orders of magnitude lower. Each species remains dominant for an interval of potencies, until it is replaced by a different species that grows faster but starts at a lower level. The transitions can be predicted well using Eqs. (11) and (12). The locations of the transitions are proportional to log(dilution factor), meaning that a 60x will be equivalent to a 30c, a 200x like a 100c, and so on.

The strengths of this model are its generality—it works the same regardless of what the actual structural ingredient turns out to be—and its power to explain a complex clinical practice from simple starting assumptions. The model may not apply if the mechanism turns out to be 'non-local,' ie does not involve discrete

information-carrying units (eg coherence or quantum entanglement[2,10,12]) or, obviously, if remedies are ultimately proved to be mere placebos or markers that support the ritual of healer–client interaction. The great weakness of the model is that it is inspired solely by clinical conventions with no direct experimental support. Still, we have provided a new way to think about the dilution–succussion cycle, which could some day suggests experiments to test the model.

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