Homeopathy (2009) 98, 35–39

doi:10.1016/j.homp.2008.10.003, available online at http://www.sciencedirect.com

ORIGINAL PAPER

Homeopathic drug selection using Intuitionistic Fuzzy Sets

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Abstract: **Using intuitionistic fuzzy set theory, Sanchez's approach to medical diagnosis has been applied to the problem of selection of single remedy from homeopathic repertorization. Two types of Intuitionistic Fuzzy Relations (IFRs) and three types of selection indices are discussed. I also propose a new repertory exploiting the benefits of soft-intelligence.** *Homeopathy* (2009) **98**, 35–39.

Keywords: Intuitionistic fuzzy set; Intuitionistic fuzzy relation; Homeopathic drug selection; Automated repertorization; Automated decision making; Homeopathic software

Introduction

The notion of intuitionistic fuzzy sets (IFSs) was introduced by Atanassov[1] in 1986. It includes both fuzzy sets[2] and vague sets[3] as special cases. Fuzzy sets are IFSs but the converse is not necessarily true. Whereas vague sets have been proven to be the same as IFSs.[4] IFSs have been found useful in diverse areas of science and technology,[5] for instance, IFSs have been applied to logic programming,[6,7] medical diagnosis, decision making problems[8] and microelectronic fault analysis.[9]

Homeopathy has been called the third-most commonly used system of healing on the globe. For that reason alone it deserves serious attention from the scientific community. In homeopathic practice, use of repertory is inevitable for a successful application. This very tedious process of repertorization of a patient's case has now been facilitated by commercially available software such as Macrep and RADAR. But such software, instead of suggesting a single remedy, leave the practitioner with a range of homeopathic medicines from which to choose, and this varies depending on the analysis strategy used. This is an intrinsic limitation of repertorial work, as alluded to by Kent himself:

> "... method [of repertorizing] is the mechanical method ... There is an artistic method that omits the mechanical, and is better, but all are not prepared to use it. ... Symptoms must be judged as to their value as characteristics in relation to the patient ... They must be valued in proportion as they relate to the patient ... Any remedy correctly worked out, ... should be perceived to agree with, and to fit, the patient ..."[10]

Hence there is a need, once the mechanical repertorization is complete, to include some automated decision in software systems to suggest the final single homeopathic remedy i.e. simillimum. Such an automation of drug selection does not replace the homeopathic practitioner on two counts: first that the gradation and choice of symptoms lies in his/her hands and secondly, the Intuitionistic Homeopathic Knowledge (described later), elicited from the experience and provings of homeopathic practitioners and researchers. This automation is an attempt to capture and standardize the process of finding the remedy which is most similar to the disease picture of a patient. It does not, by any means, replace the fundamental Hahnemannian philosophy of finding the simillimum.

In the present paper, I modify Sanchez's method of medical diagnosis[11,12] using IFS theory. After a detailed description, an illustrative example is given as an application of the method. I also discuss two types of compositions and three selection indices besides proposing a mechanism for accumulating systematic homeopathic knowledge and hence a new repertory.

Preliminaries

Definition 2.1[1]

Let X be a nonempty fixed set. An intuitionistic fuzzy set A is an object of the form $A = \{\langle x, \mu(x), \gamma(x)\rangle : x \in X\}$ where μ and γ are degrees of membership and

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Received 17 November 2007; revised 16 August 2008; accepted 8 October 2008

non-membership of each $x \in X$, respectively, and $0 \leq \mu(x) + \nu(x) \leq 1$ for each $x \in X$.

For the sake of brevity, we shall use the notation $A = \langle \mu_A, \gamma_A \rangle$ instead of $A = \{\langle x, \mu_A, \gamma_A \rangle | x \in X\}$.

The number $\pi_A(x) = 1 - (\mu_A(x) + \gamma_A(x))$ is called hesitation of IFS A, which may relate to either membership value or non-membership value or both.

Definition 2.2[1]

Let X be nonempty set and A, B IFS's in X, given as $A = <\mu_A, \gamma_A>$, $B = <\mu_B, \gamma_B>$. Then

1. $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$;
2. $A = B$ iff $A \subseteq B$ and $B \subseteq A$;
3. $\overline{A} = \{\langle x, \gamma_A(x), \mu_A(x) \rangle : x \in X\}$;
4. $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x) \rangle : x \in X\}$;
5. $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x) \rangle : x \in X\}$.

Definition 2.3[4]

An Intuitionistic Fuzzy Relation (IFR) R from a set X to another set Y is an IFS in $X \times Y$ i.e.

$$R = \{\langle (x, y), \mu_R(x, y), \gamma_R(x, y) \rangle : x \in X, y \in Y\},$$

where $\mu_R : X \times Y \to [0, 1]$ and $\gamma_R : X \times Y \to [0, 1]$ satisfy the condition

$$(\forall (x, y) \in X \times Y)(\mu_R(x, y) + \gamma_R(x, y) \leq 1).$$

An IFR R from X to Y is denoted by R_{XY}.

Definition 2.4

Let Q_{XY} and R_{YZ} be two IFRs. The max-average composition RoQ is the intuitionistic fuzzy relation from X to Z, defined by the membership and non-membership functions, respectively, as:

$$\mu_{RQ}(x, z) = \bigvee_y \left[\frac{1}{2}(\mu_Q(x, y) + \mu_R(y, z))\right] \text{ and}$$

$$\gamma_{RQ}(x, z) = \bigwedge_y \left[\frac{1}{2}(\gamma_Q(x, y) \vee \gamma_R(y, z))\right]$$

$$\forall (x, z) \in X \times Z \text{ and } \forall y \in Y.$$

Definition 2.5[5]

Let Q_{XY} and R_{YZ} be two IFRs. The max-min-max composition RoQ is the intuitionistic fuzzy relation from X to Z, defined by the membership and the non-membership functions, respectively, as:

$$\mu_{RQ}(x, z) = \bigvee_y [\mu_Q(x, y) \wedge \mu_R(y, z)] \text{ and}$$

$$\gamma_{RQ}(x, z) = \bigwedge_y \left[\gamma_Q(x, y) \vee \gamma_R(y, z)\right]$$

$$\forall (x, z) \in X \times Z \text{ and } \forall y \in Y.$$

Sanchez's scheme in terms of IFSs

Using IFSs I generalize and adapt Sanchez's scheme for medical diagnosis to the problem of selection of homeopathic simillimum. In a given homeopathic case, suppose S is the set of symptoms, D set of possible homeopathic medicines, and P the set of patients. Analogous to Sanchez's notion of "Medical Knowledge" we define Intuitionistic Homeopathic Knowledge (IHK for short) as an intuitionistic fuzzy relation K from the set of symptoms S to the set of drugs D (i.e., on $S \times D$) which reveals the degree of association and the degree of non-association between symptoms and drugs.

Method

The methodology of intuitionistic fuzzy drug selection comprises following main steps:

1. Determination of symptoms of the patient and their associations/non-association to his/her by routine case-taking. Thus mathematically, a patient is an IFS, say A, on the set of symptoms S. This IFS would help to construct an IFR C_{PS}, relating patient to symptoms.
   It is customary to underline symptoms reported by patients with more underlinings indicating stronger or more characteristic symptoms. Assigning association and non-association degrees to each symptom is a generalization of 'underlining'. For example if a practitioner uses traditional underlinings, the association degree of one underline may be fixed as 0.3, two underlines as 0.6 and three underlines as 1. Adding more grades would allow finer analysis.
   Degrees of association and non-association degrees may be interpreted variously. Below, first number of the pair is the degree to which a patient thinks the symptom to be important and the second is the degree of importance attached to it by the practitioner. For instance, the patient may attach great significance to his aching knee but for practitioner it is just a particular physical symptom, not a generality or modality and therefore relatively unimportant. The generality of method includes the classical case as a special one as follows:
   Since

   $$\mu(x) + \gamma(x) \leq 1, \forall x \in X,$$

   this allows us to select non-association $\gamma(x)$ as

   $$\gamma(x) \leq 1 - \mu(x).$$

   This implies that if the traditional analysis is used one should use the mechanical formula for the non-association i.e.

   $$\gamma(x) = 1 - \mu(x). \quad (1)$$

   In this case one is dealing with a subclass of IFS i.e. fuzzy sets and, in fact, under-utilizing this method. While using Eq. (1), we should remember that it is debatable if an association of (say) 33% automatically implies a two-third non-association.
2. Codification of homeopathic knowledge as an Intuitionistic Fuzzy Relation could be achieved by the help of an

expert homeopath or some other dependable source of homeopathic knowledge, proposed by Rutten *et al.*[13] If the 3 grade system used by repertories is applied it may be handled as in case of patient-symptom underlinings in Step 1 above. This IFR from S to D of homeopathic knowledge is denoted as K_{SD}.

3. Determination of patient-drug relational strength through composition of intuitionistic fuzzy relations. The max-min-max or max-average composition R_{PD} of C_{PS} with the IFR K_{SD} denoted by $R = KoC$ (*R*esult = *K*nowledge applied to *C*ase) signifies the patient-drug relation as an IFS on D with the membership and non-membership functions, respectively, given by

$$\mu_R(d) = \bigvee_{s\in S}[\mu_A(s) \wedge \mu_R(s,d)] \text{ and}$$

$$\gamma_R(d) = \bigwedge_{s\in S}[\gamma_A(s) \vee \gamma_R(s,d)]$$

$$\forall d \in D.$$

The choice of composition method lies with the user. If user's knowledge of homeopathic case is to be considered, then Definition 2.4 above should be employed. If one decides to depend on expert knowledge only, then choice of Definition 2.5 is more correct.

4. Computation of selection index S_K for the final decision. Any one of the following selection indices may be used:

$$S_K^1 = \mu_K - \gamma_K \cdot \pi_K$$

$$S_K^2 = \mu_K - \left(\frac{\gamma_K + \pi_K}{2}\right)$$

$$S_K^3 = \left(\frac{\mu_K + \gamma_K}{2}\right) - \pi_K$$

S_K^2 is more discriminatory than S_K^1. Whereas S_K^3 is firmer on giving judgement and seldom yields a tied decision. Hence, the use of 'hesitations' is rareley required using this formula S_K^3.

5. In case of a tied decision in Step 4 'hesitations' are used and the drug with least 'hesitations' is selected.

Generalized Method

This method can be extended to a finite number of patients as follows: Let there be n patients p_i, $i = 1, 2, \ldots, n$. Thus $p_i \in P$. Let K_{SD} be an IFR on $S \times D$ and construct an IFR C_{PS} from the set of patients to the set of symptoms S. The composition R of IFRs K and C ($R = KoC$) describes the state of patients p_i in terms of the diagnosis as an IFR from P to D given by the membership and non-membership functions, respectively, as:

$$\mu_R(p_i,d) = \bigvee_{s\in S}[\mu_C(p_i,s) \wedge \mu_K(s,d)] \text{ and}$$

$$\gamma_R(p_i,d) = \bigwedge_{s\in S}[\gamma_C(p_i,s) \vee \gamma_K(s,d)]$$

or if the max-aveage composition is used, then

$$\mu_R(p_i,d) = \bigvee_{s\in S}\left[\frac{1}{2}(\mu_C(p_i,s) + \mu_R(s,d))\right] \text{ and}$$

$$\gamma_R(x,z) = \bigwedge_{s\in S}\left[\frac{1}{2}(\gamma_C(p_i,s) + \gamma_K(s,d))\right]$$

$$\forall_{p_i} \in p \text{ and } d \in D.$$

Accumulation of homeopathic knowledge

For a given K and C, the relation $R = KoC$ can be computed. Conversely, from the knowledge of C and K, one may also compute a refined version of the IFR K (knowledge) such that the following hold true:

1. selection index is greatest, and
2. the equality $R = KoC$ is retained.

This refined version of K will be a more significant IFR attributing higher degrees of association and lower degrees of non-association of symptoms as well as lower degrees of hesitation to the drugs. Thus this is one of many possible approaches towards Accumulated Intuitionistic Homeopathic Knowledge. From a refined version of K, one may infer drugs from symptoms in the sense of a paired value, one being the degree of association and other the degree of non-association. If the homeopath is not satisfied with the results, K is modified. This improvement of K over a period of time would establish a rubric in a repertory which would be the compilation of all such K's. A computer-based knowledge acquisition system could be used for this purpose. This system could use the method presented in this work to accumulate homeopathic knowledge, more precisely, the symptom-drug associations and non-associations, from different practitioners (Figure 1).

It is important to note that this flow chart and the loop based upon expert opinion is not part of this IFS method presented in this paper, instead it is the mechanism to validate IHK which if gathered and compiled would yield a new repertory in which entries, for example, may look as follows (the subscript numbers are association as perceived by the patient and non-association as rated by the practitioner respectively):

Heat, foot, soles, uncovers them

$\text{Am} - \text{c}_{(.4,.5)}$, $\text{Calc} - \text{c}_{(.8,.2)}$, $\text{Cham}_{(.8,.1)}$, $\text{Con}_{(.2,.2)}$, $\text{Lyc}_{(.8,.1)}$, $\text{Nux} - \text{v}_{(.5,.3)}$, $\text{Puls}_{(.6,.3)}$, $\text{Sep}_{(.8,.1)}$, $\text{Sil}_{(.5,.2)}$, $\text{Sulph}_{(.5,.1)}$

Illustrative example

To illustrate the method, consider the following example of a real patient (patient's personal details anonymised):

Suppose the patient is John whose symptoms were recorded by routine case-taking practice. A pair of values was attached to each symptom: first value showing the strength of association of that symptom with the patient and the other showing non-association as perceived by the practitioner. The symptoms, the practitioner decided to include, looked something like this:

John had undergone a stomach operation and since then had been suffering from vomiting and retching (.3,.3). He has a life long complaint of coryza (.6,.2). He

becomes angry when his opinion is refuted or disagreed with (.4,.1). He avoids bathing (.7,.2). He complains of soles burning specially in the bed (.8,.1). His stool is first hard and then liquid (.9,0).

The case was repertorized using Kent's and Boericke's, repertories. Following rubrics were considered:

s_1 = [Kent] [Mind] Anger, irascibility: Contradiction, from
s_2 = [Kent] [Generalities] Bathing: Dread of
s_3 = [Kent] [Extremities] Heat: Foot: Sole: Uncovers them
s_4 = [Boericke] [Stomach] Vomiting & retching: Cause: Post-operative (laparotomy)
s_5 = [Boericke] [Nose] Coryza: Coryza, with chronic tendency
s_6 = [Kent] [Stool] Hard: First: Then fluid

Normal repertorization gave the following ten drugs as the most likely to be indicated:

Calc-c, Puls, Lyc, Am-c, Nux-v, Sil, Sep, Sulph, Cham, Con

Now we proceed to refine this result, using IHK. The rubrics/symptoms are termed arbitrarily $s_1, \ldots, s_6$ and the set of symptoms $S = \{s_1, s_2, s_3, s_4, s_5, s_6\}$.

The IFR C_{PS} is as follows:

Conclusion

To establish Homeopathy as a true science, systematization and standardization are prerequisites. A repertory of homeopathic symptoms and medicines is one of the most important sources of the practice and research in this science. Hence one reads now and then calls to develop new repertories based on Bayesian methods e.g. likelihood ratio.[13] Complementary to a new repertory is a method of repertorization which exploits its full benefit.

A two-pronged approach has been presented in this paper. A scheme to develop a new repertory, establishing the associations and non-associations of a drug with a given symptom has been presented. And a method to fully exploit the benefits of such a new repertory has been presented. For this purpose IFS theory has been used to adapt Sanchez's approach for medical diagnosis[11,12] to the problem of homeopathic drug selection. Three different selection indices and two types of IFR compositions have been discussed. The method is described in detail and applied to an example case. Besides providing a methodology, we also propose an automated mechanism to accumulate homeopathic knowledge in a systematic manner. This may help to realize the idea of a reliable repertory.[13,14]

C_{PS}	s_1	s_2	s_3	s_4	s_5	s_6
John	(.4,.1)	(.7,.2)	(.8,.1)	(.3,.3)	(.6,.2)	(.9,0)

The IFR K_{SD} (IHK) is as follows:

K_{SD}	Calc-c	Puls	Lyc	Am-c	Nux-v	Sil	Sep	Sulph	Cham	Con
s_1	(.8,.2)	(.6,.3)	(.8,.1)	(.4,.5)	(.5,.3)	(.5,.2)	(.8,.1)	(.5,.1)	(.8,.1)	(.2,.2)
s_2	(.4,.1)	(.5,.3)	(.8,.1)	(.3,.5)	(.9,0)	(.7,.1)	(.6,.3)	(.4,.2)	(.2,.2)	(.5,.3)
s_3	(.6,.3)	(.7,.1)	(.4,.6)	(.5,.2)	(.5,.3)	(.9,0)	(.3,.2)	(.6,.2)	(.3,.2)	(.5,.4)
s_4	(.7,.2)	(.6,.1)	(.8,0)	(.7,.3)	(.6,.1)	(.8,.1)	(.3,.1)	(.6,.3)	(.4,.3)	(.2,.7)
s_5	(.8,.1)	(.5,.1)	(.3,.6)	(.7,.2)	(.5,.5)	(.3,.5)	(.4,.4)	(.5,.1)	(.1,.2)	(.4,.3)
s_6	(.7,.2)	(.7,.2)	(.5,.3)	(.2,.6)	(.5,.3)	(.7,.1)	(.4,.4)	(.7,.1)	(.5,0)	(.7,.2)

Using max-average composition, $R = KoC$ is computed as:

R_{PD}	Calc-c	Puls	Lyc	Am-c	Nux-v	Sil	Sep	Sulph	Cham	Con
John	(.8,.1)	(.8,.1)	(.8,.1)	(.7,.2)	(.8,.1)	(.9,.1)	(.7,.1)	(.8,.1)	(.7,0)	(.8,.1)

$S_K^i; i = 1, 2, 3$ are:

	Calc-c	Puls	Lyc	Am-c	Nux-v	Sil	Sep	Sulph	Cham	Con
S_K^1	.8	.6	.8	.4	.4	.4	.8	.5	.8	.1
S_K^2	.7	.4	.7	.1	.3	.3	.7	.3	.7	−.2
S_K^3	.5	.4	.4	.4	.2	.1	.4	−.1	.4	−.4

S_K^1 and S_K^2 do not produce any substantial difference about the decision. Both indices give Calc-c, Lyc, Sep and Cham as equal candidates, $S_K^1 = 0.8 = S_K^2$ for all of them. To resolve this we calculate the hesitations for each drug, based upon R_{PD}, as follows:

Calc-c	Puls	Lyc	Am-c	Nux-v	Sil	Sep	Sulph	Cham	Con
.1	.1	.2	.2	.1	.1	.3	.2	.3	.1

Since least hesitation among the four candidate drugs is that of Calc-c, this is decided to be the chosen drug. Note that S_K^3 unequivocally delivers the same judgement without the hesitation calculation.

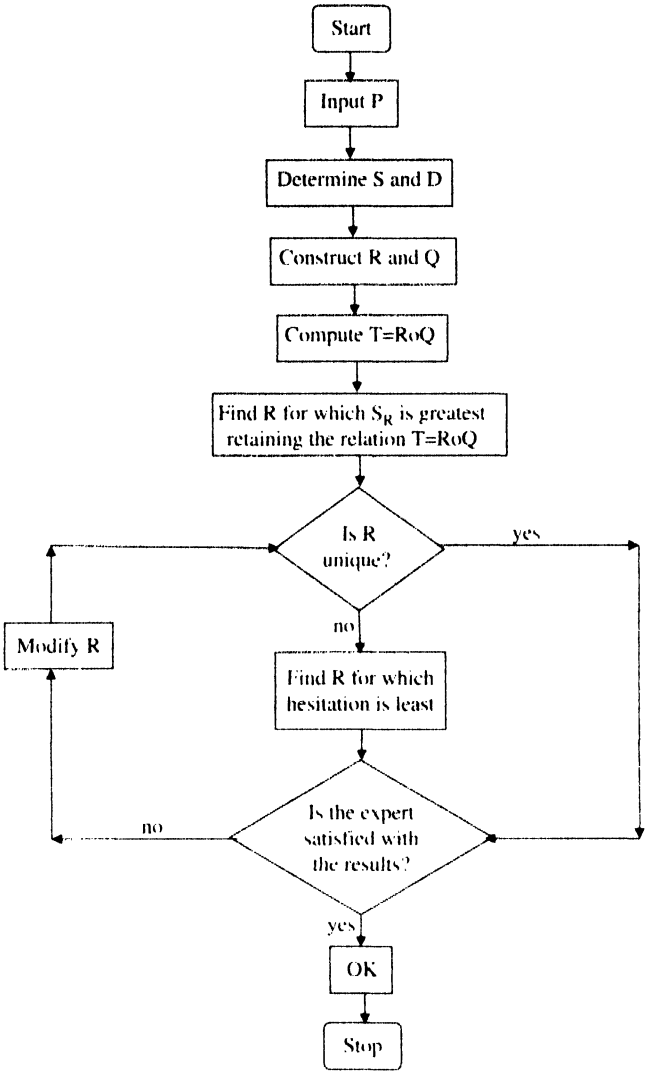

Figure 1 Flowchart to refine Intuitionistic Fuzzy Relation (*K*) for association of symptoms with homeopathic medicines.

Acknowledgement

The author wishes to gratefully acknowledge the valuable suggestions of an anonymous referee and the editor.

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